TIME-FREQUENCY METHODS FOR STRUCTURAL HEALTH MONITORING

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ABSTRACT

The ability to effectively detect and classify damage in complex materials and structures is an important problem in the area of structural health monitoring (SHM). The goal is to provide indicators about the presence, location, type, or severity of damage in a structure of interest. In this paper, we review two stochastic damage classification schemes which are based on the use of time-frequency techniques. The first method utilizes the matching pursuit decomposition (MPD) to construct cross-term free time-frequency representations (TFRs) of the structural data, with classification performed based on correlations in the time-frequency plane. The second method relies on using hidden Markov models (HMMs) to model time-frequency damage features extracted from structural data using the MPD, and classification is performed in a Bayesian framework. In both cases, the MPD is employed with time-frequency-scale dictionaries composed of highly localized Gaussian functions. Results are presented from an example application to the classification of fatigue-induced crack damage in an aluminum lug-joint specimen, and the utility of the techniques is discussed.

Index Terms— Structural health monitoring, damage detection, time-frequency analysis, matching pursuit decomposition, hidden Markov models

1. INTRODUCTION

Structural health monitoring (SHM) [1–3] deals with the design and deployment of damage identification systems for the complex materials and structures of interest in civil, mechanical, and aerospace applications. An excellent survey of the problems of SHM and current research can be found in [1, 2]. Damage detection and classification techniques strive to provide indicators about the presence, location, size, or severity of structural damage. The purpose of this article is to discuss some recent developments [4–11] in the use of time-frequency methods for damage classification in SHM applications.

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An important feature of the damage classification problem is uncertainty. The mechanism of damage nucleation and the damage evolution process are both stochastic in nature. In addition, information collected via measured sensor data is often plagued by the high degree of variability encountered in real-world SHM problems. The data may be strongly influenced, for example, by changes in temperature, geometry or configuration, sensor characteristics, and material variability. One of the key challenges in the development of effective damage detection and classification algorithms is to design a strategy which can account for and be robust to this uncertainty. Statistical techniques [12–14] are an indispensable class of methods for exploring this uncertainty, and have consequently received much attention in the SHM literature [15–17].

Another critical aspect of the problem is in the choice of signal model to use. Material wave-physics can be characterized by dispersive or time-varying phenomena, with structures often behaving akin to wave-guides. An appropriate model here, therefore, is one which can allow analysis of signals with spectral content that may vary with time. Importantly, from the SHM perspective, boundary conditions are usually altered by the presence of damage. Under these conditions, the natural tool to employ for fully exploiting the non-stationary signal structure is time-frequency analysis [18, 19].

In this paper, we review two stochastic damage classification schemes based on the use of time-frequency techniques. The first method utilizes the matching pursuit decomposition (MPD) [18, 20] to construct cross-term free time-frequency representations (TFRs) of the structural data, with classification performed based on correlations in the time-frequency plane. The second method relies on using hidden Markov models (HMMs) [21, 22] to model time-frequency damage features extracted from structural data using the MPD, and classification is performed in a Bayesian framework. In both cases, the MPD is employed with time-frequency-scale dictionaries composed of highly localized Gaussian functions. The data used for learning model parameters may be obtained either from physically based modeling (virtual sensing) or from real sensor measurements. We present results from an example application to the classification of fatigue-induced crack damage in an aluminum lug-joint specimen, and discuss the utility of the techniques.

2. TIME-FREQUENCY ANALYSIS BASED DAMAGE CLASSIFICATION

2.1. Feature Extraction with Matching Pursuit Decomposition

The objective of feature extraction is to condense the data gathered by the measurement process into a form suitable for further analysis and processing, but with minimum loss in the information of interest. Typically, this amounts to a reduction in the dimensionality or number of degrees of freedom in the data via a parameterized representation of some form or another. Decomposition of the measured signals into a (weighted) linear combination of chosen basis functions or "atoms" is a popular choice of a parameterized representation, for example.

For the purpose of damage classification, we are interested in extracting those features from the observed data which succinctly capture the essentials of the material wave-physics and yield maximum discriminatory information between the various damage classes in question. We argue that features based on time-frequency representations may be ideally suited for this task. This is because, as mentioned earlier, material wave-physics can be well characterized by time-varying spectral content, which is potentially affected by the presence of damage.

A wide variety of tools are now available for achieving time-frequency type decompositions, such as wavelet transforms and many redundant approximations (see [18] for an excellent survey). Our approach relies on the use of the matching pursuit decomposition (MPD) [18, 20]. The MPD is a pursuits-based signal approximation that is known to be much more versatile than its orthogonal counterparts. Unlike wavelets, where the time-frequency tiling is predetermined and fixed, the MPD can be used to effect time-frequency decompositions which are highly adaptive. Importantly, the added flexibility does not come at a very high price: the MPD remains an efficient $O(N \log N)$ method (where N is the signal size).

The MPD employs a greedy algorithm to compute representations for signals in terms of basis functions chosen from a redundant dictionary. For a signal $s(t) \in \mathbf{L}^2(\mathbb{R})$, the unique L-term MPD representation $s_L(t)$, computed iteratively, is of the form

$$s(t) \approx s_L(t) = \sum_{l=0}^{L-1} \alpha_l \, g_{\gamma_l}(t), \tag{2.1}$$

where α_l are the expansion coefficients and $g_{\gamma_l}(t)$ are the basis functions selected from a dictionary $\mathcal{D}$. For algorithmic details, we refer the reader to [20]. If the dictionary $\mathcal{D}$ is designed well, the error in the approximation can be made small using only a few number of terms L.

In our work, we utilize the MPD with a time-frequency-scale dictionary composed of highly localized Gaussian atoms [20], which are time-frequency (TF) shifted and scaled versions of a basic Gaussian atom $g(t) = Ce^{-t^2/2}$. The Gaussian atoms have optimal time-frequency resolution properties and are therefore well-suited to analyze structural data.[1] The dictionary atoms are given by

$$g_\gamma(t) = C_\gamma \, e^{-\kappa^2 \, (t-\tau)^2} \cos\left(2\pi\nu t\right), \tag{2.2}$$

where τ is the time-shift, ν is the frequency-shift, κ is the (positive) scaling parameter, and C_γ is an appropriate normalizing constant. Each Gaussian atom $g_\gamma(t) \in \mathcal{D}$ is thus fully characterized by an element $\gamma = \{\tau, \nu, \kappa\}$ from the set $\Gamma = \mathbb{R}^2 \times \mathbb{R}^+$. Together with the expansion coefficients α_l, the time-frequency-scale parameters in γ_l can be used as "features" to leverage the differences in the time-frequency signatures of the signals from the various damage classes.

An indispensable signal-analysis tool afforded by the MPD is the cross-term free time-frequency representation (MPD-TFR) [20], given by

$$\mathscr{E}_s(t, f) = \sum_{l=0}^{L-1} |\alpha_l|^2 \, \mathrm{WD}_{g_{\gamma_l}}(t, f), \tag{2.3}$$

where WD denotes the Wigner distribution [19]. Figure 2.1 shows example MPD-TFRs of data from an aluminum lug-joint sample with and without crack damage. Note the marked difference in the time-frequency structure between the damaged and undamaged cases.

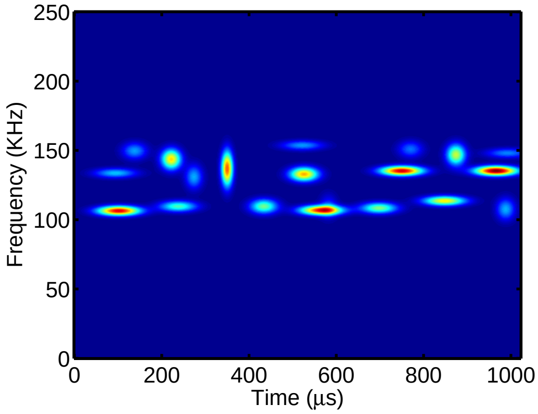
2.2. MPD Time-Frequency Representation based Damage Classifier

The MPD time-frequency based damage classification algorithm [4, 10] utilizes the MPD-TFRs to classify damage by computing correlations in time-frequency space. The classification is carried out in two steps. The first step is to "train" the MPD classifier using a representative set of training signals from the damage classes of interest. Specifically, MPD is performed on the training signals and the resulting MPD-TFRs are used to construct template TFRs characterizing the time-frequency structure of each damage class. Let M denote the number of damage classes, and $s_{k,m}(t)$ the MPD representations of the training signals for each class, $m = 1, 2, \ldots, M$, $k = 1, 2, \ldots, K_m$, where K_m is the number of training signals used for class m. The template TFRs for each class are computed by averaging the MPD-TFRs of the training signals from that class as

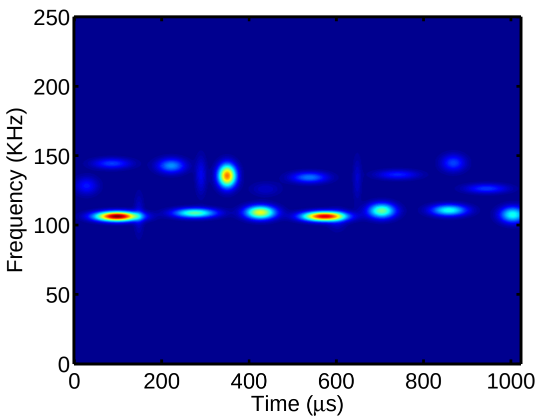
$$\mathscr{E}_{s_m}(t, f) = \frac{1}{K_m} \sum_{k=1}^{K_m} \mathscr{E}_{s_{k,m}}(t, f), \tag{2.4}$$

and stored for use in the classification.

[1] Dictionaries based on real measured data have also been studied [9, 10], and the resulting decompositions have been shown to be extremely parsimonious.



(a) MPD-TFR (undamaged)



(b) MPD-TFR (with 6 mm crack)

Fig. 2.1. MPD-TFRs of data for an aluminum lug-joint sample (a) undamaged, and (b) with 6 mm crack damage.

Classification is then performed based on the use of two-dimensional correlations with the template TFRs. The strength of the TFR correlations in the time-frequency plane is used to quantify how similar or dissimilar a given signal is to known members of each class. Suppose that we have a test signal $r(t)$ which needs to be classified to one of M different classes (structural conditions). We first obtain its MPD-TFR $\mathscr{E}_r(t,f)$, and then assign $r(t)$ to class m^*, given by

$$m^* = \underset{m\in\{1,\ldots,M\}}{\text{argmax}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathscr{E}_r(t,f)\mathscr{E}_{s_m}(t,f)\,dt\,df. \quad (2.5)$$

Note that the TFRs are normalized appropriately before computing the correlations.

The statistical accuracy of the estimated template TFRs is greater when a large amount of data is available for training, leading to a more robust classifier. In addition to training on multiple signals from the same experiment, it is desirable to train on measurements from different experiments but pertaining to the same damage and boundary conditions.

2.3. Hidden Markov Model based Damage Classifier

A hidden Markov model (HMM) [21, 22] is a probabilistic model used for modeling sequential data. Consider a length-T observation sequence $\mathbf{y} = \{y_1,\ldots,y_T\}$. The HMM defines a probability distribution over $\mathbf{y}$ by invoking another sequence of unobserved (hidden) discrete variables $\mathbf{x} = \{x_1,\ldots,x_T\}$ known as 'states'. The model imposes (a) Markov dynamics on the sequence of hidden states, and (b) independence of the observations y_n from all other variables given x_n. Suppose that the number of distinct states is N, with the state variables x_n assuming values from the alphabet $\{1,\ldots,N\}$. The model is then parameterized by the $N\times 1$ initial state distribution vector π whose ith element is the probability $p(x_1 = i)$, the $N\times N$ state-transition matrix A whose (i,j)th element is $p(x_{n+1} = j|x_n = i)$, and the state-dependent observation density B whose jth element is $b_j(y_n) = p(y_n|x_n = j)$; together the model parameters are denoted as the set $\boldsymbol{\theta} = \{\pi, A, B\}$. In a discrete HMM, the observations y_n are discrete. In a continuous HMM, the observations are continuous and the observation density B is often modeled using a Gaussian mixture model (GMM).

The HMM based damage classification approach [5, 6, 11] relies on the statistical characterization of the time-frequency state dynamics of data from each damage class with a Markov random process. Specifically, the HMM is used to statistically model the time-frequency features extracted from structural data by the MPD. The observation sequences $\mathbf{y}$ are of length $T = L$ and comprised of the four-dimensional vectors $y_n = \{\alpha_n, \tau_n, \nu_n, \kappa_n\}$, $n = 1,\ldots,L$ of the extracted MPD atoms (in a discrete HMM, the observations are quantized). The features from each structural condition (damage class) are modeled with a separate HMM.

The training data available from each damage class is used to learn the parameters of the corresponding HMM. Given the training signals $\mathbf{y}$, we compute the maximum-likelihood (ML) [12, 13] estimate of the parameters $\boldsymbol{\theta}$ as

$$\boldsymbol{\theta}_{\text{ML}} = \arg\max_{\boldsymbol{\theta}} \log p(\mathbf{y}|\boldsymbol{\theta}) \quad (2.6)$$

using the Baum-Welch algorithm [21, 22], a special case of the expectation-maximization (EM) algorithm [12, 13] which iteratively maximizes the likelihood of the training data. The details of the HMM re-estimation procedure can be found in [11, 22]. Classification is then performed in a Bayesian framework [12, 13]: a test signal $\mathbf{y}'$ is classified to damage class m', given by

$$m' = \arg\max_{m\in\{1,\ldots,M\}} \log p(\mathbf{y}'|\boldsymbol{\theta}^m_{\text{ML}}), \quad (2.7)$$

where $\boldsymbol{\theta}^m_{\text{ML}}$ denotes the parameters of the HMM associated with the mth damage class.

3. APPLICATION TO THE CLASSIFICATION OF FATIGUE DAMAGE

We now present an example application of the time-frequency analysis methods to the classification of damage in a real structure of interest. The problem considered here is that of classifying fatigue-induced crack damage in a aluminum lug-joint specimen (lug-joints are ubiquitous in aircraft landing gear and are known damage hotspots). Data is collected from fatigue testing of a 0.25 in. thick polished Al 2024 T351 lug-joint sample. Surface-mounted piezoelectric sensors are used to actuate and measure response to a tone burst signal of central frequency 130 KHz. Sensor data is recorded for five different structural conditions (damage classes): class 1 (0 mm crack), class 2 (6 mm crack), class 3 (8 mm crack), class 4 (10 mm crack), and class 5 (12 mm crack). Altogether, 1500 signals are collected (300 for each damage class).

We apply the MPD-TFR and HMM based damage classifiers to this problem. Time-frequency feature extraction is first performed on the data using $L = 20$ iterations of MPD with a dictionary composed of about 12.5 million Gaussian atoms. Half of the data is used for training and half for testing. For the discrete HMM classifier, the MPD features are discretized using vector quantization. $N = 3$-state HMMs are utilized (a variation Bayesian (VB) learning technique for estimating the number of HMM states is discussed in [6]), with parameters estimated using 20 iterations of the Baum-Welch algorithm.

Tables 3.1 shows the average correct classification rates obtained for the fatigue damage classification problem above using the MPD-TFR, discrete HMM, and continuous HMM classifiers. From the results shown in Table 3.1, we see that

Classification method	Average correct classification rate
MPD-TFR	92.9%
Discrete HMM (64 codes)	88.6%
Discrete HMM (128 codes)	90.8%
Discrete HMM (256 codes)	92.6%
Continuous HMM	94.4%

Table 3.1. Average correct classification rates obtained for the fatigue damage classification problem using the time-frequency based damage classifiers.

the performance of all the time-frequency based damage classifiers is good (the average correct classification rates are near 90%). The performance of the discrete HMM classifier improves with the number of codes due to lower quantization error. Note that the classification results shown here are obtained using data from a single sensor. Integration of information collected by multiple distributed sensors via Bayesian sensor fusion [10, 11] has been reported to result in further improvements in classification performance.

4. DISCUSSION

Time-frequency analysis coupled with appropriate stochastic modeling techniques can be a powerful tool for the investigation of damage in complex materials and structures. Our experience shows that, under controlled conditions, and when adequate data is available, it is possible to achieve very good damage classification performance.

Real-world damage identification, however, can be a much more difficult task. Firstly, our present classification methods assume that training data is available from each class of interest. The approach also requires that the training data be representative of the damage being considered, i.e., it should account for the variability expected due to changes in operational and environmental conditions. The supervised learning approaches employed further impose that the given training data is labeled. Second, data is assumed to be available in sufficient quantity so as to be able to reasonably estimate the parameters of the relevant models. Moreover, test data can only be classified to one of the classes learned. In order to add a more realistic flavor to all this (for example, for situations in which we require that data can possibly be classified to previously unseen classes, or when the data is scarce), we are currently investigating some advanced machine learning techniques, such as active and multi-task learning [23–25] that provide for greater flexibility and efficiency in the modeling framework.

More generally, the overall SHM philosophy pursued here—that the tasks of damage detection, localization, and quantification can be cast into some sort of a damage classification problem—while a reasonable starting point, is probably too constraining for many applications. For successful real-world SHM and prognosis, the sensing and signal processing must be integrated with information from physically based modeling. All this forms the subject of current research and results will be reported at a later date.

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